DUAL CONTROLLER BY CORRECTIONS VECTOR AND ITS EXTENSION TO SYSTEMS WITH COLOURED NOISE

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INTRODUCTION

The dual control plays two roles: (1) to bring the system to the desired reference and (2) to produce the signals required to identify the system. An optimal dual control strategy is not implementable on real time. As a consequence, several sub-optimal dual control strategies have been developed. Some of them are the active sub-optimal dual controller (ASDC, Wittenmark (1)) and the innovations dual controller (IDC, Milito et al (2)). In this work a new sub-optimal dual control strategy is proposed and compared to the two ones previously mentioned. The simulations have stressed the aspect of the transient behaviour of the diverse strategies. This is done in order to verify the following hypothesis: If the dual control weighs both the control task and the identification task. Then this type of control may eventually improve the transient nature of the estimation and control.

DUAL CONTROLLER BY CORRECTIONS VECTOR (DCCV)

System with white noise and stochastic parameters.

The type of system to be controlled is:

$$y(t) = \phi(t)\ \theta(t) + e(t) \tag{1}$$

where

$\phi(t)=[u(t-1)\ldots u(t-n)u(t-n-1)y(t-1)\ldots y(t-n)]$: observations vector, u is the control and y is the output; $\theta(t)=[b_o b_1\ldots b_n - a_1\ldots - a_n]^T$:

stochastic parameters vector;
e(t):white gaussian noise, zero mean and variance σ^2.

The vector $\theta(t)$ is assumed to be described by a first order markovian process:

$$\theta(t+1) = \Phi\theta(t) + L(t) \tag{2}$$

where

Φ is a known (2n+1)x(2n+1) matrix.
L(t) is a (2n+1)x1 gaussian white noise vector, independent of e(t), with zero mean and covariance matrix R.

The identification algorithm is a Kalman filter which considers eq.(1) as the output equation and equation (2) as the state equation. Thus, it is obtained:

$$\begin{aligned}\hat{\theta}(t+1) &= \Phi\hat{\theta}(t)+K(t)(y(t) - \phi(t)\hat{\theta}(t))\\ K(t) &= \Phi P(t)\phi(t)^T(\phi(t)P(t)\phi(t)^T + \sigma^2)^{-1}\\ P(t+1) &= (\Phi-K(t)\phi(t))P(t)\Phi^T + R\end{aligned} \tag{3}$$

where $\hat{\theta}(t)$ is the expected value of $\theta(t)$ conditioned by the observations, and P(t) is the estimation error covariance matrix.

The proposed control strategy is obtained by minimizing the objective function:

$$J = E\{[(y(t) - y_R(t))^2 - \lambda\|V_c(t)\|^2]/I_t\} \tag{4}$$

I_t are all the previous observations, $y_R(t)$ is the reference and $V_c(t)$ is the corrections vector defined by:

$$V_c(t) = K(t)\nu(t) = K(t)(y(t)-\phi(t)\hat{\theta}(t)) \tag{5}$$

The minimizations of all the functionals which appear in this paper are done respect with u(t-1).

The weighting factor assigned to the squared norm of the corrections vector in eq.(4) forces the controller to increase the variance of the parameters information, implying dual characteristics in the controller.

The minimization of the functional described by eq.(4) is reduced to finding the roots of a fifth order polynomial determining therefore a non-explicit control law.

The proposed controller may be considered as an extension of the Wittenmark's controller (1) and the Milito's controller (2).

The ASDC (1) minimizes the functional:

$$J = E\{[(y(t)-y_R(t))^2 + \lambda P_{b_o}(t+1)]/I_t\} \tag{6}$$

where $P_{b_o}(t)$ is the estimation error variance of the parameter $b_o(t)$. Notice that the minimization of the functional described by eq. (5) is equivalent to minimize:

$$J = E\{[(y(t)-y_R(t))^2 + \lambda \mathrm{tr}P(t+1)]/I_t\} \tag{7}$$

where the estimation error of all the parameters are considered.

By the other side the IDC (2) considers the minimization of:

$$J = E\{[(y(t) - y_R(t))^2 - \lambda\nu^2(t)]/I_t\} \tag{8}$$

where $\nu(t)$ are the innovations used to feed the Kalman Filter. The controller by corrections vector considers the innovations multiplied by a time varying factor which depends on K(t). This provides for a better correction in the identification.

System with coloured noise and constant parameters.

Consider the system described by:

$$A(q^{-1})\ y(t) = B(q^{-1})\ u(t-1) + C(q^{-1})e(t) \tag{9}$$

where the operator q^{-1} is defined by

$$q^{-1}x(t)= x(t-1)$$

and
$$A(q^{-1}) = 1+a_1 q^{-1} +\ldots+a_n q^{-n}$$
$$B(q^{-1}) = b_0+b_1q^{-1} +\ldots+b_n q^{-n}$$
$$C(q^{-1}) = 1+c_1q^{-1} +\ldots+c_n q^{-n}$$

The polynomial $C(q^{-1})$ has all its roots inside the unit circle.

Consider now the signal $u_R(t-1)$ which produces the desired output $y_R(t)$ in the model given by eq. (9) when perturbations are not present ($e(t)=0$):

$$A(q^{-1})y_R(t) = B(q^{-1})u_R(t-1) \qquad (10)$$

Substracting (10) from (9) it is obtained:

$$A(q^{-1})\hat{y}(t)=B(q^{-1})(u(t-1)-u_R(t-1))+C(q^{-1})e(t) \qquad (11)$$

where $\hat{y}(t)= y(t)-y_R(t)$

Equation (11) can be transformed in the following one step predictor (Aström, (3)):

$$\hat{y}(t)=e(t)+\frac{G(q^{-1})}{C(q^{-1})}\hat{y}(t-1)+\frac{B(q^{-1})}{C(q^{-1})}u(t-1)+ +(-\frac{A(q^{-1})}{C(q^{-1})}y_R(t)) \qquad (12)$$

where $G(q^{-1}) = g_1 + g_2q^{-1} +\ldots+g_nq^{1-n}$

Defining the generalized output (Clarke and Gawthrop, (4)):

$$Y(t) = \hat{y}(t) + \beta u(t-1) \qquad (13)$$

the following predictive equation is obtained

$$Y(t) = [\frac{G(q^{-1})}{C(q^{-1})}y(t-1) + \frac{D(q^{-1})}{C(q^{-1})}u(t-1) - \frac{A(q^{-1})}{C(q^{-1})}y_R(t)]+e(t) \qquad (14)$$

where

$$D(q^{-1})=B(q^{-1})+\beta C(q^{-1})=d_0+d_1q^{-1}+\ldots+d_nq^{-1}$$

It is well known (Aström and Wittenmark (5) Ljung (6)) that making $C(q^{-1})=1$, defining

$$\theta(t)=[d_1\ldots d_ng_1\ldots g_n\ldots g_n-a_1\ldots-a_n]^T$$

$$\phi(t)=[u(t-2)\ldots u(t-n-1)y(t-1)\ldots y(t-n)y_R(t-1)\ldots y_R(t-n)]$$

and applying the control law:

$$u(t-1)=\frac{1}{\hat{d}_0}[y_R(t) - \phi(t)\hat{\theta}(t)] \qquad (15)$$

where $\hat{\theta}(t)$ are estimated parameters by least squares and $\hat{d}_0$ is a fixed value, the controller found converges to the optimal controller which minimizes:

$$J = E\{[(y(t) - y_R(t))^2+ \lambda_2u^2(t-1)]/I_t\}$$

if $\beta = \lambda_2/b_0$. When the system is a minimal phase one, λ_2 can be equal to zero.

In this work, the use of dual sub-optimal control strategies is proposed. Especially the corrections vector one applied to the predictive equation (14). The following two arguments support the idea: a) Predictive equation like (14) where the output prediction ($Y(t)$) is based upon available observations at time "t-1", allow the utilization of a Kalman filter as the identifier.
b) If strategies like equation (15) constitute algorithms for which it has been proved that $\hat{\theta}=\theta$ (θ are the true controller parameters) is an stable convergence point ((5),(6)), then for a dual sub-optimal controller, $\hat{\theta}=\theta$ will be also an stable convergence point. This is true because in the convergence state when the covariances matrix $P(t)$ tends to zero, the dual sub-optimal strategies tend to laws in the form of the equation (15).

SIMULATION RESULTS

The following dual sub-optimal control strategies were applied to zero order, first order and second order systems perturbed with white noise:

a) Innovations dual controller (IDC)
b) Active sub-optimal dual controller (ASDC)
c) Dual controller by corrections vector (DCCV)

The self tuning regulator was applied to the second order system. The zero order system has been treated before (a) by Aström and Wittenmark in (7) who used an optimal adaptive control strategy (OACS), (b) by Wittenmark in (1), who used the ASDC and (c) by Milito et al in (8), who used the IDC.

A first order system perturbed by coloured noise was controlled by the self-tuning regulator and by the dual controller by corrections vector. Finally, the DCCV was applied to a non minimal phase system.

The results obtained through simulations were compared over the following performance indices in order to analyse the transient response:

Acumulated cost $\equiv V(N)= \sum_{n=0}^{N} [y(n)-y_R(n)]^2$

Energy spending in the control signal $\equiv$

$$C(N) = \sum_{n=0}^{N} u(n)^2$$

Variance of the parameters estimation error $\equiv$ diag $P(N)$.

Accumulated cost by step $\equiv$

$$\overline{V}(N,t) = \frac{1}{n+1}\sum_{n=t}^{t+N} [y(n)-y_R(n)]^2$$

Energy spending by step $\equiv$

$$\overline{C}(N,t) = \frac{1}{n+1}\sum_{n=t}^{t=N} u(n)^2$$

The quantities just defined are averaged over N_R realizations.

Sistems Perturbed by White Noise

a) Zero order system

Equation: $y(t)=b(t)u(t-1)+e(t)$

where:
$$e(t)\sim N(0,0.25) \qquad (16)$$
$$b(t+1)=0.9\ b(t)+V(t) \qquad (17)$$
$$V(t)\sim N(0,1) \qquad (18)$$
$$E\ V(t).e(t) =0 \qquad (19)$$

$\hat{b}(0)=b(0)=s$ (20)
$s \sim N(0,1)$ (21)
$P(0) = 1$
$y_R(t)=1.0$
$N_R=1000$

Table 1 summarizes the results obtained from the simulations for zero order systems. In this case the proposed controller coincides with the ASDC.

b) First order system

Equation: $y(t)=b(t)u(t-1)+a(t)y(t-1)+e(t)$

where (16), (17), (18), (19), (20) and (21) are satisfied and

$a(t) = 0.9$
$\hat{b}(0) = 10^{-4}$
$\hat{a}(0) = 10^{-4}$
$P(0) = 1000\ I$
$y(-1) = 0$
$y_R(t) = 10.0$
$N_R = 500$

The results for this case are shown in table 2.

c) Second order systems

Equation: $y(t)=0.28446\ u(t-1)+0.26972\ u(t-2)$

$+1.846602\ y(t-1)-0.8521438y(t-2)+e(t)$

where (16) is satisfied and

$P(0) = 1000\ I$
$\theta_i(0) = 10^{-4}$
$y(-1) = 0$
$y(-2) = 0$
$y_R(t) = 0$
$N_R = 500$

Table 3 shown the results obtained for this case.

Systems Perturbed by Coloured Noise

In this case the initial values were:

$\hat{\theta}_i(0) = 10^{-3}$
$P(0) = 100\ I$
$y(-1) = 0$
$N_R = 500$

Case 1: The system considered is:

$$y(t)=0.5\ u(t-1)+y(t-1)+e(t)+0.1e(t-1)$$

where $e(t) \sim N(0,0.5)$

The reference model to be followed with minimum variance is:

$$y_R(t) = 0.4\ u_R(t-1)+ 0.6\ y_R(t-1)$$

The results obtained are shown in table 4.

Case 2: Now we consider a non minimal phase system:

$$y(t)=u(t-1)+1.05u(t-2)+y(t-1)+e(t)+0.7e(t)$$

where $e(t) \sim N(0,0.5)$ and $N_R=500$

The reference model to be followed is described by:

$$y_R(t) = 0.2(u_R(t-1)+ 0.5u_R(t-2))+0.7y_R(t-1)$$

The results obtained for this case are shown in table 5.

ANALISIS OF RESULTS AND CONCLUSIONS

The tables mentioned summarize the results obtained for the different controllers. The selected values of λ (dual controllers) are those which allowed the least accumulated costs (V(N)) for each one of the considered dual controllers.

These values of λ were found through simulations. It is pertinent to remark the importance of selecting the appropiate value of λ. An over dimensioned value of λ entails a too active controller neglecting the control task. On the contrary if λ is too low the controller will be too cautions. The "appropiate" value of λ dependes on each particular case and up to the present there is not theoretical tool which provide guidelines for the choosing of λ a priori.

From the results some conclussions can be drawn about the efficiency of the controllers. The proposed controller presents the lowest cost and therefore the best transient adaptation. The self tuning regulator presents a good transient behavior if $\hat{b}_0=b_0$, being highly sensitive to the variations of this parameter. The results obtained in the simulations with coloured noise and non-minimal phase systems allow to conclude that the proposed controller may be a new alternative in industrial applications. Evidently its major disadvantage compared to the self tuning controller is its higher computational cost.

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TABLE 1 Dual Control applied to a zero order system (OACS: optimal adaptive control strategy; IDC: Innovations dual controller DCCV dual controller by corrections vector; ASDC: active sub-optimal dual controller)

	OACS			IDC(λ=0.6)			DCCV=ASDC(λ=0.4)		
HORIZON	V(N)	C(N)	P(N)	V(N)	C(N)	P(N)	V(N)	C(N)	P(N)
5	6.7338	.82910	1.6448	6.9664	1.1316	1.7166	6.6031	.84854	1.6395
10	11.086	1.6226	1.6876	11.484	2.1681	1.7687	10.995	1.6881	1.6639
30	27.299	4.6564	1.7248	28.700	6.2102	1.7634	27.099	4.9510	1.6861
60	51.601	9.1672	1.7224	54.649	12.211	1.7749	51.492	9.8123	1.6804
100	84.043	15.053	1.7194	89.304	20.214	1.7565	84.084	16.306	1.6779
200	164.88	30.239	1.7155	174.67	40.237	1.7725	165.05	32.602	1.6748
300	246.18	45.330	1.7041	261.00	60.250	1.7461	246.11	48.837	1.6771
400	327.05	60.189	1.7843	346.97	80.206	1.7593	327.79	65.101	1.6768

TABLE 2 Dual Control Applied to a First Order System

	IDC(λ=0.7)				ASDC(λ=0.004)				DCCV(λ=0.01)			
HORIZON	$\bar{V}$	$\bar{C}$	$\bar{P}_b$	$\bar{P}_a$	$\bar{V}$	$\bar{C}$	$\bar{P}_b$	$\bar{P}_a$	$\bar{V}$	$\bar{C}$	$\bar{P}_b$	$\bar{P}_a$
0→10	82.001	9.0848	350.20	186.40	71.736	4.5727	259.73	188.18	66.323	5.4776	249.75	187.05
11→20	15.404	7.4000	1.6475	1.1E-2	18.524	4.0316	1.7850	1.5E-2	12.966	2.2537	1.7846	1.6E-2
21→30	7.4230	3.7793	1.6108	8.6E-4	8.8640	1.3020	1.6847	1.4E-3	8.0738	1.2841	1.6886	1.6E-3
31→40	7.5510	3.7630	1.5704	3.8E-4	7.6530	1.1846	1.6572	6.0E-4	8.0892	1.2265	1.6662	4.8E-4
41→50	7.5780	3.8773	1.5352	2.5E-4	7.2780	1.1654	1.6406	3.6E-4	7.5186	1.1578	1.6191	3.4E-4
181→200	6.9837	3.5392	1.5398	5.0E-5	7.7474	1.1540	1.5688	7.1E-5	6.6183	1.0482	1.5998	7.0E-5
V(50)	1281.57				1212.29				1096.03			

TABLE 3 Dual Control and Self-Tuning Controller Applied to a Second Order System with Constant Parameters

	SELF-TUNNING CONTROLLER				IDC		ASDC		DCCV	
	$\hat{b}_o$ = 0.28446		$\hat{b}_o$ = 0.4		λ = .8		λ = .5		λ = .5	
HORIZON	$\bar{V}$	$\bar{C}$	$\bar{V}$	$\bar{C}$	$\bar{V}$	$\bar{C}$	$\bar{V}$	$\bar{C}$	$\bar{V}$	$\bar{C}$
0→10	689.20	2.90E5	3.6E11	1.6E12	420.61	4.08E3	1.09E4	5.72E4	236.58	1.39E4
11→20	.30105	6.45E5	4.1E19	4.3E20	2.71E4	1.81E7	2.98E3	1.30E7	206.88	3.85E5
21→30	.26151	2.23E5	1.4E20	7.4E22	191.40	2.03E7	6.6260	5.86E6	.41275	2.08E5
31→40	.25481	7.66E4	4.4E16	2.6E22	2.7046	7.08E6	2.7878	2.19E6	.30261	7.16E4
41→50	.25482	2.66E4	3.2E13	9.2E21	.39038	2.45E6	2.4009	8.57E5	.26273	2.49E4
181→200	.25110	220.42	5.36E6	5.0E15	.25210	238.89	.25032	244.45	.24785	218.28
V(50)	7591.92		1.8E21		2.7788E5		1.4945E5		4681.01	

TABLE 4 Self-Tuning Controller and DCCV Applied to a First Order System with Coloured Noise (Constant Parameters)

	SELF-TUNING CONTROLLER			DCCV				
	$\bar{V}(N,t)$			$\bar{V}(N,t)$				
HORIZON	$\hat{b}_0=.35$	$\hat{b}_0=.5$	$\hat{b}_0=.7$	$\lambda_1=.001$	$\lambda_1=.003$	$\lambda_1=.01$	$\lambda_1=.03$	$\lambda_1=.04$
0→10	4.34E7	47.838	2927.2	70.795	67.268	57.395	46.008	48.219
11→20	8.20E4	.56581	1.8462	13.857	9.9277	3.2411	1.4086	1.2431
21→30	9.40E3	.53516	.72545	.56968	.53873	.55718	.52291	.58186
31→40	3.0E3	.53072	.63443	.53881	.54986	.51684	.52948	.53063
41→50	1.35E3	.52291	.58089	.52764	.53905	.51700	.51972	.52274
192→203	.48991	.50443	.49927	.50972	.49220	.51066	.49175	.49268
V(50)	4.8E8	547.76	3.22E4	933.67	855.51	679.66	535.90	559.19

TABLE 5 DCCV Applied to a Non-minimal Phase System (Constant Parameters)

	$\bar{V}(N,t)$					
	$\lambda_2=0.2$			$\lambda_2=0.4$		
HORIZON	$\lambda_1=.001$	$\lambda_1=.006$	$\lambda_1=.007$	$\lambda_1=.001$	$\lambda_1=.006$	$\lambda_1=.007$
0→10	89.372	142.246	499.75	86.723	76.539	108.23
11→20	799.03	2227.0	1681.7	300.56	295.17	338.84
21→30	25.709	24.446	15.365	6.6703	26.635	1.6243
31→40	2.0553	1.1239	1.0270	1.4807	1.0379	1.0454
41→50	1.3832	.87244	.83875	1.0716	.97756	.93220
192→203	.80898	.79333	.80069	.89817	.88283	.89375
V(50)	9264.9	2.40E4	2.2E4	4051.8	4080.1	4506.6